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Can QFT on Moyal-Weyl spaces look as on commutative ones?

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Abstract

We sketch a natural affirmative answer to the question based on a joint work [11] with J. Wess. There we argue that a proper enforcement of the "twisted Poincaré" covariance makes any differences $(x-y)^\mu$ of coordinates of two copies of the Moyal-Weyl deformation of Minkowski space like undeformed. Then QFT in an operator approach becomes compatible with (minimally adapted) Wightman axioms and time-ordered perturbation theory, and physically equivalent to ordinary QFT, as observables involve only coordinate differences.

1 Introduction: twisting Poincaré group and Minkowski spacetime

In the last decade a broad attention has been devoted to the construction of QFT on Moyal-Weyl spaces, perhaps the simplest examples of noncommutative spaces. These are characterized by coordinates $\hat{x}^\mu$ fulfilling the commutation relations

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1}$$

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where $\theta^{\mu\nu}$ is a constant real antisymmetric matrix. For present purposes $\mu = 0, 1, 2, 3$ and indices are raised or lowered through multiplication by the standard Minkowski metric $\eta_{\mu\nu}$, so as to obtain a deformation of Minkowski space. We shall denote by $\widehat{\mathcal{A}}$ the algebra"of functions on Moyal-Weyl space", i.e. the algebra generated by $\mathbf{1}, \hat{x}^\mu$ fulfilling (1). For $\theta^{\mu\nu} = 0$ one obtains the algebra $\mathcal{A}$ generated by commuting x^μ.

Clearly (1) are translation invariant, but not Lorentz-covariant. As recognized in [5, 18, 13, 14], they are however covariant under a deformed version of the Poincaré group, namely a triangular noncocommutative Hopf $*$-algebra H obtained from the UEA $U\mathcal{P}$ of the Poincaré Lie algebra $\mathcal{P}$ by *twisting* [9][1]. This means that (up to isomorphisms) H and $U\mathcal{P}$ (extended over the formal power series in $\theta^{\mu\nu}$) are the same $*$-algebras, have the same counit ε, but different coproducts $\Delta, \hat{\Delta}$ related by

$$\Delta(g) \equiv \sum_I g^I_{(1)} \otimes g^I_{(2)} \;\longrightarrow\; \hat{\Delta}(g) = \mathcal{F}\Delta(g)\mathcal{F}^{-1} \equiv \sum_I g^I_{(\hat{1})} \otimes g^I_{(\hat{2})} \tag{2}$$

for any $g \in H \equiv U\mathcal{P}$. The antipodes are also changed accordingly. The socalled twist $\mathcal{F}$ is not uniquely determined, but what follows does not depend on its choice. The simplest is

$$\mathcal{F} \equiv \sum_I \mathcal{F}^{(1)}_I \otimes \mathcal{F}^{(2)}_I := \exp\left(\tfrac{i}{2}\theta^{\mu\nu} P_\mu \otimes P_\nu\right). \tag{3}$$

P_μ denote the generators of translations, and in (2), (3), we have used Sweedler notation; $\sum_I$ may denote an infinite sum (series), e.g. $\sum_I \mathcal{F}^{(1)}_I \otimes \mathcal{F}^{(2)}_I$ comes out from the power expansion of the exponential. A straightforward computation gives

$$\hat{\Delta}(P_\mu) = P_\mu \otimes \mathbf{1} + \mathbf{1} \otimes P_\mu = \Delta(P_\mu), \qquad \hat{\Delta}(M_\omega) = M_\omega \otimes \mathbf{1} + \mathbf{1} \otimes M_\omega + P[\omega, \theta] \otimes P \neq \Delta(M_\omega),$$

where we have set $M_\omega := \omega^{\mu\nu} M_{\mu\nu}$ and used a row-by-column matrix product on the right. The left identity shows that the Hopf P-subalgebra remains undeformed and equivalent to the abelian translation group $\mathbb{R}^4$. Therefore, denoting by $\triangleright, \hat{\triangleright}$ the actions of $U\mathcal{P}, H$ (on $\mathcal{A}$ $\triangleright$ amounts to the action of the corresponding algebra of differential operators, e.g. P_μ can be identified with $i\partial_\mu := i\partial/\partial x^\mu$), they coincide on first degree polynomials in $x^\nu, \hat{x}^\nu$,

$$P_\mu \triangleright x^\rho = i\delta^\rho_\mu = P_\mu \hat{\triangleright} \hat{x}^\rho, \qquad M_\omega \triangleright x^\rho = 2i(x\omega)^\rho, \qquad M_\omega \hat{\triangleright} \hat{x}^\rho = 2i(\hat{x}\omega)^\rho, \tag{4}$$

and more generally on irreps (irreducible representations); this yields the same classification of elementary particles as unitary irreps of $\mathcal{P}$. But $\triangleright, \hat{\triangleright}$ differ on products of coordinates, and more generally on tensor products of representations, as $\triangleright$ is extended by the rule $g \triangleright (ab) = (g_{(1)} \triangleright a)(g_{(2)} \triangleright b)$ involving $\Delta(g)$ (the rule reduces to the usual Leibniz rule for $g = P_\mu, M_{\mu\nu}$), whereas $\hat{\triangleright}$ is extended as at the lhs of

$$g \hat{\triangleright} (\hat{a}\hat{b}) = \sum_I (g^I_{(\hat{1})} \hat{\triangleright} \hat{a})(g^I_{(\hat{2})} \hat{\triangleright} \hat{b}) \quad \Leftrightarrow \quad g \triangleright_\star (a \star b) = \sum_I (g^I_{(\hat{1})} \triangleright_\star a) \star (g^I_{(\hat{2})} \triangleright_\star b), \tag{5}$$

[1] In section 4.4.1 of [14] this was formulated in terms of the dual Hopf algebra

involving $\hat{\Delta}(g)$ and a *deformed* Leibniz rule for $M_\omega\hat{\triangleright}$. Summarizing, the H-module unital $*$-algebra $\widehat{\mathcal{A}}$ is obtained by twisting the $U\mathcal{P}$-module unital $*$-algebra $\mathcal{A}$.

Several spacetime variables. The proper noncommutative generalization of the algebra of functions generated by n sets of Minkowski coordinates x_i^μ, $i=1,2,...,n$, is the noncommutative unital $*$-algebra $\widehat{\mathcal{A}}^n$ generated by real variables $\hat{x}_i^\mu$ fulfilling the commutation relations at the lhs of

$$[\hat{x}_i^\mu, \hat{x}_j^\nu] = \mathbf{1}i\theta^{\mu\nu} \qquad \Leftrightarrow \qquad [x_i^\mu \overset{\star}{,} x_j^\nu] = \mathbf{1}i\theta^{\mu\nu}; \tag{6}$$

note that the commutators are not zero for $i \neq j$. The latter are compatible with the Leibniz rule (5), so as to make $\widehat{\mathcal{A}}^n$ a H-module $*$-algebra, and dictated by the braiding associated to the quasitriangular structure $\mathcal{R} = \mathcal{F}_{21}\mathcal{F}^{-1}$ of H.

As H is even triangular, an essentially equivalent formulation of these H-module algebras is in terms of $\star$-products derived from $\mathcal{F}$. For $n \geq 1$ denote by $\mathcal{A}^n$ the n-fold tensor product algebra of $\mathcal{A}$ and $x^\mu \otimes \mathbf{1} \otimes ...,\ \mathbf{1} \otimes x^\mu \otimes ...,...$ respectively by x_1^μ, x_2^μ,... Denote by $\mathcal{A}_\theta^n$ the algebra obtained by endowing the vector space underlying $\mathcal{A}^n$ with a new product, the $\star$-product, related to the product in $\mathcal{A}^n$ by

$$a \star b := \textstyle\sum_I (\overline{\mathcal{F}}_I^{(1)} \triangleright a)(\overline{\mathcal{F}}_I^{(2)} \triangleright b), \tag{7}$$

with $\overline{\mathcal{F}} \equiv \mathcal{F}^{-1}$. This encodes both the usual $\star$-product within each copy of $\mathcal{A}$, and the "$\star-$tensor product" algebra [2, 3]. As a result one finds the isomorphic $\star$-commutation relations at the rhs of (6) (this follows from computing $x_i^\mu \star x_j^\nu$, which e.g. for the specific choice (3) gives $x_i^\mu x_j^\nu + i\theta^{\mu\nu}/2$) and that $\widehat{\mathcal{A}}^n, \mathcal{A}_\theta^n$ are isomorphic H-module unital $*$-algebras, in the sense of the equivalence (5). More explicitly, on analytic functions f, g (7) reads $f(x_i) \star g(x_j) = \exp[\frac{i}{2}\partial_{x_i}\theta\partial_{x_j}]f(x_i)g(x_j)$, and must be followed by the indentification $x_i = x_j$ *after* the action of the bi-pseudodifferential operator $\exp[\frac{i}{2}\partial_{x_i}\theta\partial_{x_j}]$ if $i\!=\!j$. It should be extended to functions in $L^1 \cap \mathbb{F}L^1$ in the obvious way using their Fourier transforms $\mathbb{F}$. In the sequel we shall formulate the noncommutative spacetime only in terms of $\star$-products and construct QFT on it replacing all products of functions and/or fields with $\star$-products.

Let $a_i \in \mathbb{R}$ with $\sum_i a_i = 1$. An alternative set of real generators of $\mathcal{A}_\theta^n$ is:

$$\xi_i^\mu := x_{i+1}^\mu - x_i^\mu, \quad i=1,...,n-1, \qquad X^\mu := \textstyle\sum_{i=1}^n a_i x_i^\mu \tag{8}$$

It is immediate to check that $[X^\mu \overset{\star}{,} X^\nu] = \mathbf{1}i\theta^{\mu\nu}$, so X^μ generate a copy $\mathcal{A}_{\theta,X}$ of $\mathcal{A}_\theta$, whereas $\forall b \in \mathcal{A}_\theta^n$

$$\xi_i^\mu \star b = \xi_i^\mu b = b \star \xi_i^\mu \qquad \Rightarrow \qquad [\xi_i^\mu \overset{\star}{,} b] = 0, \tag{9}$$

so ξ_i^μ generate a $\star$-central subalgebra $\mathcal{A}_\xi^{n-1}$, and $\mathcal{A}_\theta^n \sim \mathcal{A}_\xi^{n-1} \otimes \mathcal{A}_{\theta,X}$. The $\star$-multiplication operators $\xi_i^\mu \star$ have the same spectral decomposition on all $\mathbb{R}$ (including 0) as multiplication opertaors $\xi^\mu \cdot$ by classical coordinates, which make up a space-like, or a null, or

a time-like 4-vector, in the usual sense. Moreover, $\mathcal{A}_\xi^{n-1}, \mathcal{A}_{\theta,X}$ are actually H-module subalgebras, with

$$
\begin{aligned}
& g \hat{\triangleright} a = g \triangleright a && a \in \mathcal{A}_\xi^{n-1}, \quad g \in H \\
& g \hat{\triangleright} (a \star b) = \left(g_{(1)} \triangleright a\right) \star \left(g_{(2)} \hat{\triangleright} b\right), && b \in \mathcal{A}_\theta^n,
\end{aligned}
\tag{10}
$$

i.e. *on $\mathcal{A}_\xi^{n-1}$ the H-action is undeformed*, including the related part of the Leibniz rule. [By (10) $\star$ can be also dropped]. All ξ_i^μ are translation invariant, X^μ is not.

2 Revisiting Wightman axioms for QFT and their consequences

As in Ref. [17] we divide the Wightman axioms [16] into a subset (labelled by **QM**) encoding the quantum mechanical interpretation of the theory, its symmetry under space-time translations and stability, and a subset (labelled by **R**) encoding the relativistic properties. Since they provide minimal, basic requirements for the field-operator framework to quantization we try to apply them to the above noncommutative space keeping the QM conditions, "fully" twisting Poincaré-covariance R1 and being ready to weaken locality R2 if necessary.

QM1. The states are described by vectors of a (separable) Hilbert space $\mathcal{H}$.

QM2. The group of space-time translations $\mathbb{R}^4$ is represented on $\mathcal{H}$ by strongly continuous unitary operators $U(a)$. The spectrum of the generators P_μ is contained in $\overline{V}_+ = \{p_\mu : p^2 \geq 0, p_0 \geq 0\}$. There is a unique Poincaré invariant state Ψ_0, the *vacuum state*.

QM3. The fields (in the Heisenberg representation) $\varphi^\alpha(x)$ [α enumerates field species and/or $SL(2,\mathbb{C})$-tensor components] are operator (on $\mathcal{H}$) valued tempered distributions on Minkowski space, with Ψ_0 a *cyclic* vector for the fields, i.e. $\star$-polynomials of the (smeared) fields applied to Ψ_0 give a set $\mathcal{D}_0$ dense in $\mathcal{H}$.

We shall keep QM1-3. Taking v.e.v.'s we define the *Wightman functions*

$$
\mathcal{W}^{\alpha_1,\ldots,\alpha_n}(x_1,...,x_n) := (\Psi_0, \varphi^{\alpha_1}(x_1) \star ... \star \varphi^{\alpha_n}(x_n)\Psi_0)\,,
\tag{11}
$$

which are in fact distributions, and (their combinations) the *Green's functions*

$$
G^{\alpha_1,\ldots,\alpha_n}(x_1,...,x_n) := (\Psi_0, T[\varphi^{\alpha_1}(x_1) \star ... \star \varphi^{\alpha_n}(x_n)]\Psi_0)
\tag{12}
$$

where also *time-ordering* T is defined as on commutative space (even if $\theta^{0i} \neq 0$),

$$
T[\varphi^{\alpha_1}(x) \star \varphi^{\alpha_2}(y)] = \varphi^{\alpha_1}(x) \star \varphi^{\alpha_2}(y) \star \vartheta(x^0 - y^0) + \varphi^{\alpha_2}(y) \star \varphi^{\alpha_1}(x) \star \vartheta(y^0 - x^0)
$$

(ϑ denotes the Heavyside function). This is well-defined as $\vartheta(x^0-y^0)$ is $\star$-central.

QM1-3 (alone) imply exactly the same properties as on commutative space:
W1. Wightman and Green's functions are translation-invariant tempered distributions and therefore may *depend only on the* ξ_i^μ:

$$\begin{aligned}\mathcal{W}^{\alpha_1,\dots,\alpha_n}(x_1,\dots,x_n) &= W^{\alpha_1,\dots,\alpha_n}(\xi_1,\dots,\xi_{n-1}),\\ \mathcal{G}^{\alpha_1,\dots,\alpha_n}(x_1,\dots,x_n) &= G^{\alpha_1,\dots,\alpha_n}(\xi_1,\dots,\xi_{n-1}).\end{aligned} \tag{13}$$

W2. (**Spectral condition**) The support of the Fourier transform $\widetilde{W}$ of W is contained in the product of forward cones, i.e.

$$\widetilde{W}^{\{\alpha\}}(q_1,\dots q_{n-1}) = 0, \qquad \text{if } \exists j: \quad q_j \notin \overline{V}_+. \tag{14}$$

W3. $\mathcal{W}^{\{\alpha\}}$ fulfill the **Hermiticity and Positivity** properties following from those of the scalar product in $\mathcal{H}$.

R1. (*Untwisted* **Lorentz Covariance**) $SL(2,\mathbb{C})$ is represented on $\mathcal{H}$ by strongly continuous unitary operators $U(A)$, and under extended Poincaré transformations $U(a,A)=U(a)\,U(A)$

$$U(a,A)\,\varphi^\alpha(x)\,U(a,A)^{-1} = S^\alpha_\beta(A^{-1})\,\varphi^\beta(\Lambda(A)x+a), \tag{15}$$

with S a finite dimensional representation of $SL(2,\mathbb{C})$.

In *ordinary* QFT as a consequence of QM2,R1 one finds
W4. (**Lorentz Covariance of Wightman functions**)

$$\mathcal{W}^{\alpha_1..\alpha_n}(\Lambda(A)x_1,\dots,\Lambda(A)x_n) = S^{\alpha_1}_{\beta_1}(A)..S^{\alpha_n}_{\beta_n}(A)\mathcal{W}^{\beta_1..\beta_n}(x_1,\dots,x_n). \tag{16}$$

In particular, Wightman (and Green) functions of scalar fields are Lorentz invariant.

R1 needs a "twisted" reformulation **R1**$_\star$, which we defer. Now, however R1$_\star$ will look like, it should imply that $W^{\{\alpha\}}$ are $SL_\theta(2,\mathbb{C})$ tensors (in particular invariant if all involved fields are scalar). But, as the $W^{\{\alpha\}}$ are to be built only in terms of ξ_i^μ and other $SL(2,\mathbb{C})$ tensors (like $\partial_{x_i^\mu}$, $\eta_{\mu\nu}$, γ^μ, etc.), which are all annihilated by $P_\mu\triangleright$, $\mathcal{F}$ will act as the identity and $W^{\{\alpha\}}$ will transform under $SL(2,\mathbb{C})$ as for $\theta=0$. Therefore **we shall require W4 also if** $\theta\neq 0$ as a temporary substitute of R1$_\star$.

The simplest sensible way to formulate the $\star$-analog of locality is
R2$_\star$. (**Microcausality or locality**) The fields either $\star$-commute or $\star$-anticommute at spacelike separated points

$$[\,\varphi^\alpha(x) \overset{\star}{,} \varphi^\beta(y)\,]_\mp = 0, \qquad \text{for } (x-y)^2<0. \tag{17}$$

This makes sense, as space-like separation is sharply defined, and reduces to the usual locality when $\theta = 0$. Whether there exist reasonable weakenings of R$2_\star$ is an open question also on commutative space, and the same restrictions will apply.

Arguing as in [16] one proves that QM1-3, W4, R$2_\star$ are independent and compatible, as they are fulfilled by free fields (see below): the noncommutativity of a Moyal-Weyl space is compatible with R$2_\star$! As consequences of R$2_\star$ one again finds
W5. (**Locality**) if $(x_j - x_{j+1})^2 < 0$

$$\mathcal{W}(x_1, ...x_j, x_{j+1}, ...x_n) = \pm\mathcal{W}(x_1, ...x_{j+1}, x_j, ...x_n). \tag{18}$$

W6. (**Cluster property**) For any spacelike a and for $\lambda \to \infty$

$$\mathcal{W}(x_1, ...x_j, x_{j+1} + \lambda a, ..., x_n + \lambda a) \to \mathcal{W}(x_1, ..., x_j)\, \mathcal{W}(x_{j+1}, ..., x_n), \tag{19}$$

(convergence in the distribution sense); this is true also with permuted x_i's.

Summarizing: our QFT framework is based on **QM1-3, W4, R$2_\star$**, or alternatively on the constraints **W1-6** for $\mathcal{W}^{\{\alpha\}}$, exactly as in QFT on Minkowski space. We stress that this applies for all $\theta^{\mu\nu}$, even if $\theta^{0i} \neq 0$, contrary to other approaches.

3 Free and interacting scalar field

As the differential calculus remains undeformed, so remain the equation of motions of free fields. Sticking for simplicity to the case of a scalar field of mass m, the solution of the Klein-Gordon equation reads as usual

$$\varphi_0(x) = \int d\mu(p)\, [e^{-ip\cdot x} a^p + a_p^\dagger e^{ip\cdot x}\,] \tag{20}$$

where $d\mu(p) = \delta(p^2 - m^2)\vartheta(p^0)d^4p = dp^0\delta(p^0 - \omega_{\mathbf{p}})d^3\mathbf{p}/2\omega_{\mathbf{p}}$ is the invariant measure ($\omega_{\mathbf{p}} := \sqrt{\mathbf{p}^2 + m^2}$). Postulating all the axioms of the preceding section (including **R$2_\star$**), one can prove up to a positive factor the **free field commutation relation**

$$[\varphi_0(x) \stackrel{\star}{,} \varphi_0(y)] = 2 \int \frac{d\mu(p)}{(2\pi)^3}\, \sin\,[p\cdot(x-y)]\,, \tag{21}$$

coinciding with the undeformed one. Applying ∂_{y^0} to (21) and setting $y^0 = x^0$ [this is compatible with (6)] one finds **the canonical commutation relation**

$$[\varphi_0(x^0, \mathbf{x}) \stackrel{\star}{,} \dot{\varphi}_0(x^0, \mathbf{y})] = i\,\delta^3(\mathbf{x} - \mathbf{y}). \tag{22}$$

As a consequence of (21), also the n-point Wightman functions coincide with the undeformed ones, i.e. vanish if n is odd and are sum of products of 2-point functions (factorization) if n is even. This of course agrees with the cluster property W6.

A φ_0 fulfilling (24) can be obtained from (22) plugging $a^p, a^\dagger_p$ satisfying

$$\begin{array}{lll} a^\dagger_p a^\dagger_q = e^{ip\theta' q}\, a^\dagger_q a^\dagger_p, & a^p a^q = e^{ip\theta' q}\, a^q a^p, & a^p a^\dagger_q = e^{-ip\theta' q}\, a^\dagger_q a^p + 2\omega_{\mathbf{p}}\delta^3(\mathbf{p}-\mathbf{q}), \\ (\text{with } \theta' = \theta), & \text{and } [a^p, f(x)] = [a^\dagger_p, f(x)] = 0, & \end{array} \qquad (23)$$

(here $p\theta q := p_\mu \theta^{\mu\nu} q_\nu$), as adopted e.g. in [4, 12, 1]. We briefly consider the consequences of choosing $\theta' \neq \theta$ [$\theta' = 0$ gives CCR among the $a^p, a^\dagger_p$, assumed in most of the literature, explicitly [8] or implicitly, in operator [6, 7] or in path-integral approach to quantization]. One finds the non-local $\star$-commutation relation

$$\varphi_0(x) \star \varphi_0(y) = e^{i\partial_x(\theta-\theta')\partial_y} \varphi_0(x) \star \varphi_0(y) + i\, F(x-y),$$

and the corresponding (free field) Wightman functions violate W4, W6, unless $\theta' = \theta$. One can obtain (23) also by assuming nontrivial transformation laws $P_\mu \triangleright a^\dagger_p = p_\mu a^\dagger_p$, $P_\mu \triangleright a^p = -p_\mu a^p$ and extending the $\star$-product law (7) also to $a^p, a^\dagger_p$. It amounts to choosing $\theta' = -\theta$ in (23), see [11] for details; the relations define examples of deformed Heisenberg algebras covariant under a (quasi)triangular Hopf algebra H [15, 10].
Normal ordering is consistently defined as a map which on any monomial in a^p, $a^\dagger_q$ reorders all a^p to the right of all $a^\dagger_q$ adding a factor $e^{-ip\theta' q}$ for each flip $a^p \leftrightarrow a^\dagger_q$, e.g.

$$:a^p a^q: = a^p a^q, \quad :a^\dagger_p a^q: = a^\dagger_p a^q, \quad :a^\dagger_p a^\dagger_q: = a^\dagger_p a^\dagger_q, \quad :a^p a^\dagger_q: = a^\dagger_q a^p e^{-ip\theta' q}.$$

(for $\theta' = 0$ one finds the undeformed definition), and is extended to fields requiring $\mathcal{A}^n_\theta$-bilinearity. As a result, one finds that the v.e.v. of any normal-ordered $\star$-polynomial of fileds is zero, that normal-ordered products of fields can be obtained from products by the same subtractions, and **the same Wick theorem** as in the undeformed case. Applying **time-orderd perturbation theory** to an interacting field again one can heuristically derive the Gell-Mann–Low formula

$$G(x_1, ..., x_n) = \frac{\left(\Psi_0, T\left\{\varphi_0(x_1) \star ... \star \varphi_0(x_n) \star \exp\left[-i\lambda \int dy^0\ H_I(y^0)\right]\right\}\Psi_0\right)}{\left(\Psi_0, T\exp\left[-i\int dy^0\ H_I(y^0)\right]\Psi_0\right)}. \qquad (24)$$

Here φ_0 denotes the free "in" field, i.e. the incoming field in the interaction representation, and $H_I(x^0)$ is the interaction Hamiltonian in the interaction representation. By inspection one finds that the **Green functions (24) coincide with the undeformed ones** (at least perturbatively). They can be computed by Feynman diagrams with the undeformed Feynman rules. See [11] for some conclusions on these results, in striking contrast with the ones found in most of the literature.

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